Name Key

Probability

Definition

Probability is a means to measure the likelihood of the occurrence of an event. You can express probability as a real number, between 0 and 1 or as a percent. The probability of an impossible event is 0 or 0%. The probability of a certain event, which must happen, is 1 or 100%.

When you gather data by observing an event, you can calculate an ***experimental probability***. In order to calculate experimental probability of an event, use the following definition.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Example 1:

A student flipped a coin 50 times. The coin landed on heads 28 times. Find the experimental probability of having the coin land on heads.

$$P(\text{heads}) = \frac{28}{50} = 0.56 = 56\%$$

Simulations

Simulation is an effective tool for finding experimental probability when conducting an actual trial is difficult. For example, if you are going to take a 6 question true-false quiz and want to know the experimental probability of guessing exactly 2 answers out of 6 correctly. You can use simulation by flipping a coin. If heads represents a correct answer, flip the coin 6 times. Then, record the number of heads. Repeat the simulation 100 times. Divide the number of times you got 2 heads by 100.

Theoretical Probability

When you roll a die, the total possible outcomes are 1, 2, 3, 4, 5, and 6. The set of all possible outcomes is known as the **sample space**. To find a ***theoretical probability***, find the ratio of outcomes.

Definition

In a sample space that has n equally likely outcomes and an event, A, occurs m of these outcomes, then the theoretical probability of event A is $P(A) = \frac{m}{n}$

Example 2:

You select a number at random from the sample space {1, 2, 3, 4, 5}. Find the theoretical probability

***P* (the number is prime).**

Since 2, 3, and 5 are the only prime numbers in the sample space: $P(\text{the number is prime}) = \frac{3}{5} = 60\%$

Example 3: Real-World Connection

Brown is a dominant eye color for human beings. If a father and a mother each carry a gene for brown eyes and a gene for blue eyes, what is the probability of their having a child with blue eyes?

Make a table. Let B represent the dominant gene (brown eyes). Let b represent the recessive gene (blue eyes).

		Gene from mother	
		B	b
Gene from	B	BB	Bb
	b	Bb	bb

The theoretical probability that the child will be born with blue eyes is 25%, or one in four births.

Geometric Probability

Sometimes you can use areas to find theoretical probability.

Example 4:

A circular pool of radius 10 ft. is enclosed within a rectangular yard measuring 50 ft. by 100 ft. If a ball from an adjacent golf course lands at a random point within the yard, what is the probability that the ball lands in the pool?

$$
\begin{aligned}
P(E) &= \frac{\text{area of pool}}{\text{total area of yard}} \\
&= \frac{\pi r^2}{lw} \\
&= \frac{\pi(10)^2}{(50)(1000)} \\
&= \frac{100\pi}{5000} \\
&= \frac{157}{2500} \\
&\approx 0.0628
\end{aligned}
$$

The probability that a golf ball will land in the pool is about 6.28%

Extra Practice:

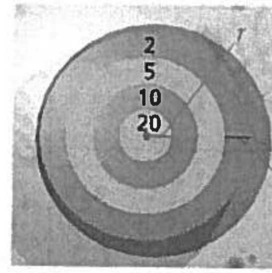
1. Find the theoretical probability of rolling a multiple of 3 with a number cube.

   $\frac{2}{6} = \frac{1}{3}$

2. Brown is a dominant eye color for human beings. If a father and mother each carry a gene for brown eyes and gene for blue eyes, what is the probability of their having a child with brown eyes?

   $\frac{1}{4}$ or 25%

3. Suppose that all points on the circular dartboard shown at the right are equally likely to be hit by a dart you have thrown.



a. Find the probability of scoring at least ten points.

$$\frac{4r^2\pi}{16r^2\pi} = \frac{1}{4} \qquad \frac{(2r)^2 \cdot \pi}{(4r)^2 \cdot \pi}$$

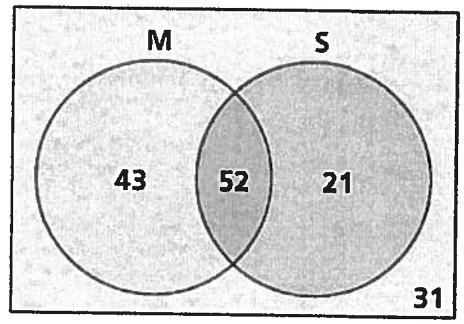
b. P (scoring 20 points).

$$\frac{r^2\pi}{16r^2\pi} = \frac{1}{16}$$

c. P (scoring 5 points). $\frac{5}{16}$

$$\frac{9r^2\pi - 4r^2\pi}{16r^2\pi} = \frac{5r^2\pi}{16r^2\pi}$$

4. In a class of 147 students, 95 are taking math (M), 73 are taking science (S), and 52 are taking both math and science. One student is picked at random. Find each probability.



a. P (taking math or science or both)

$$\frac{43+52+21}{147} = \frac{116}{147} \quad 79\%$$

b. P(not taking math)

$$\frac{21+31}{147} = \frac{52}{147} \quad 35\%$$

c. P(taking math but not science)

$$\frac{43}{147} \quad 29\%$$

d. P(taking neither math nor science)

$$\frac{31}{147} \quad 21\%$$

~ Probability Practice ~

Probability measures how likely it is for an event to occur. You can express probabilities as percents (0% to 100%), as decimals (0 to 1), or as fractions (0 to 1).

A probability of 0 or 0% means it is impossible for the event to occur.

A probability of 1 or 100% means it is definite that the event will occur.

Theoretical probability explains what SHOULD happen in a simulation.

THEORETICAL PROBABILITY: $P(event) = \dfrac{number\ of\ specific\ outcomes}{total\ number\ of\ possible\ outcomes}$

Ex) Find the theoretical probability of flipping a coin and having it land on heads.

$$P(heads) = \frac{number\ of\ sides\ with\ a\ head}{total\ number\ of\ sides} = \frac{1}{2}$$

Experimental probability is what DOES happen. It shows data from a real-life simulation.

EXPERIMENTAL PROBABILITY: $P(event) = \dfrac{number\ of\ times\ event\ occurs}{number\ of\ trials}$

Ex) Flip a coin 10 times. Find the experimental probability of the coin landing on heads.

$$P(heads) = \frac{number\ of\ flips\ coin\ showed\ heads}{total\ number\ of\ flips} = \frac{\quad}{\quad}$$

We will deal exclusively with theoretical probability in this lesson, from now on.

I. SINGLE EVENT PROBABILITY

Evaluate the following probabilities based on the roll of a single die (number cube):

1. P (2) = $\frac{1}{6}$
2. P (5) = $\frac{1}{6}$
3. P (1 or 4) = $\frac{2}{6} = \frac{1}{3}$
4. P (odd) = $\frac{3}{6} = \frac{1}{2}$
5. P (8) = $\frac{0}{6} = 0$
6. P (1-6) = $\frac{6}{6} = 1$

Evaluate the following probabilities based on pulling a card from a 52-card deck (13 of each suit):

7. P (heart) = $\frac{13}{52} = \frac{1}{4}$
8. P (spade) = $\frac{13}{52} = \frac{1}{4}$
9. P (ace) = $\frac{4}{52} = \frac{1}{13}$
10. P (2 of Diamonds) = $\frac{1}{52}$

II. MULTIPLE EVENT PROBABILITY

When the outcome of one event affects the outcome of a second event, the two events are *dependent events*. When the outcome of one event does not affect the outcome of a second event, the two events are *independent events*.

Classify each pair of events as *dependent* or *independent*:

11. Roll a number cube. Then toss a coin. Independent

12. Choose a time for a Monday dental appointment. Then choose a time for that same Monday to meet a friend for coffee. dependent

We use the word **"and"** to connect independent events. To find multiple probabilities of independent events, we **multiply** the single probabilities.

If A and B are independent events, then P(A and B) = P(A) • P(B).

Roll a die and toss a coin.

13. P (heads and 3) = $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

14. P (tails and even) = $\frac{1}{2} \cdot \frac{3}{6} = \frac{3}{12} = \frac{1}{4}$

Flip two coins.

15. P (heads and heads) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

16. P (tails and then a heads) = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

When two events cannot happen at the same time, the events are *mutually exclusive*.

Are the events mutually exclusive?

17. Rolling a 2 or a 3 on a number cube. yes cannot happen @ the same time

18. Rolling an even number or a multiple of 3 on a number cube. no can happen @ the same time w/ a 6

We use the word **"or"** to connect mutually exclusive events. To find multiple probabilities of mutually exclusive events, we add the single probabilities.

If A and B are mutually exclusive events, then P(A or B) = P(A) + P (B).

Pull a card from a deck of 52 cards:

19. P (2 or ace) = $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

20. P (heart or spade) = $\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$

Some probability problems involve replacing or not replacing an item. In these problems, we use the same notations that we use in other multiple probability problems:

$$P(A \text{ and } B) = P(A) \bullet P(B)$$
$$P(A \text{ or } B) = P(A) + P(B)$$

There are 10 marbles in a jar. Four are red, three are green, one is purple, and two are black.

21. What is the probability of choosing a red or a green marble?

$$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

22. What is the probability of choosing a red marble, replacing it, and then picking a green marble?

$$\frac{4}{10} \cdot \frac{3}{10} = \frac{12}{100} = \frac{3}{25}$$

23. What is the probability of choosing a green marble, replacing it, and then picking a black marble?

$$\frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$$

24. What is the probability of choosing a red marble, keeping it <u>(not replacing it)</u>, and then picking a green marble?

$$\frac{4}{10} \cdot \frac{3}{9} \text{ (down 1)} = \frac{12}{90} = \frac{2}{15}$$

25. What is the probability of choosing a green marble, <u>keeping</u> it, and then picking a black marble?

$$\frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$$

~ More Probability Practice ~

Find the probability of each event occurring.

For these cards, assume a normal deck of 52 cards. Assume that you return a card after you use it.

1. P (not a 7) $48/52 = 12/13$

2. P (Queen of Spades) $1/52$

3. P (not a black 3) $50/52 = 25/26$

4. P (black 7) $2/52 = 1/26$

5. P (heart) $13/52 = 1/4$

6. P (5) $4/52 = 1/13$

7. P (11) $0/52 = 0$

8. P (7 or 11) $\frac{4}{52} + \frac{0}{52} = \frac{4}{52} = 1/13$

9. P (red 5 or any 6) $\frac{2}{52} + \frac{4}{52} = \frac{6}{52} = 3/26$

10. P (3 and 5) $\frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$

11. What is the probability of tossing a coin 3 times in a row and getting 3 heads? $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

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A bag contains 36 red, 48 green, 22 yellow, and 19 purple blocks. You choose one from the bag at a time and replace it.

12. P (green) $48/125$

13. P (purple) $19/125$

14. P (not yellow) $103/125$

15. P (green and then yellow) $\frac{48}{125} \cdot \frac{22}{125} = \frac{1056}{15625}$

16. P (red or purple) $36/125 + \frac{19}{125} = \frac{55}{125} = 11/25$

17. P (purple and black) $\frac{19}{125} \cdot \frac{0}{125} = \frac{0}{15625} = 0$

A die (number cube) is tossed. Find each probability.

18. P (2 or odd) $\frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

19. P (less than 2 or greater than 4) $\frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

20. Two number cubes are tossed. What is the probability of getting a 3 and then an even number?

$\frac{1}{6} \cdot \frac{3}{6} = \frac{3}{36} = \frac{1}{12}$

A jar contains four blue marbles and two red marbles. Suppose you choose a marble at random, and do not replace it. Then you choose a second marble.

21. You select a blue marble and then a red marble. $\frac{4}{6} \cdot \frac{2}{5} = \frac{8}{30}$ $4/15$

22. Both of the marbles you select are red. $\frac{2}{6} \cdot \frac{1}{5} = 2/30$ $1/15$

23. You select a green and then a blue marble. $\frac{0}{6} \cdot \frac{4}{5} = \frac{0}{30}$ 0

24. You select a red or blue marble and then a red or blue marble. 1

$\frac{6}{6} \cdot \frac{5}{5} = \frac{30}{30}$