Linear Functions

Key

WRITING EQUATIONS FOR HORIZONTAL AND VERTICAL LINES.

- A horizontal line represents a *constant function*. Its equation will always be: $y = b$

- A vertical line represents all the places where x is a specific number. This is **not** a function and can't be solved for *y*. Its equation will always be: $x = a$

Word problems in Slope-intercept form

When a word problem involves a constant rate or speed and a beginning amount, it can be written in slope-intercept form: $y = mx + b$ To do this, recognize which number will represent *m*, the rate, and which number will represent *b*, the y-intercept.

(x) independent → time

1. An airplane 30,000 feet above the ground begins descending at the rate of 2000 feet per minute. Assume the plane continues at the same rate of descent. The plane's height and minutes above the ground are related to each other.

   Identify the variables in this situation: x= # of minutes   y= altitude

   What is the given information in this problem (find all that apply)?
   y-intercept 30,000 slope -2000 one point (___ , ___) a second point: (___ , ___)
   not needed this time

   a. Write an equation to model the situation.

   $$y = -2000x + 30000$$

   b. <u>Use your equation</u> to find the altitude of the plane after 5 minutes.

   $$y = -2000(5) + 30,000$$
   $$y = -10,000 + 30,000 = 20,000 \text{ feet}$$

2. Suppose you receive $100 for a graduation present, and you deposit it in a savings account. Then each week thereafter, you add $5 to the account but no interest is earned. The amount in the account is a function of the number of weeks that have passed.

   Identify the variables in this situation: x= # of weeks   y= amt. in acct.

   What is the given information in this problem (find all that apply)?
   y-intercept 100 slope 5 one point (___ , ___) a second point: (___ , ___)

   a. Find an equation for the amount *y* you have after *x* weeks.

   $$y = 5x + 100$$

   b. <u>Use your equation to</u> find when you will have $310 in the account.

   $$310 = 5x + 100 \qquad x = 42 \text{ wks.}$$

Word Problems in Point-slope form

When a word problem involves a constant rate or speed and gives a relationship at some
point in time between each variable, an equation can be written in point-slope form to model the relationship.

1. Marty is spending money at the average rate of $3 per day. After 14 days he has $68 left. The amount left depends on the number of days that have passed.

   Identify the variables in this situation: x= __#of days__   y= __amt in acct,__

   What is the given information in this problem (find all that apply)?

   y-intercept _____ slope __3__ one point (14 , 68) a second point: (___ , ___)

   a. Write an equation for the situation.

   $$y - 68 = 3(x - 14) \qquad y = 3x + 26$$
   $$y - 68 = 3x - 42$$

   b. Use your equation to find the amount of money he began with.

   $$y = 3(0) + 26$$
   $$y = 26$$

2. Diane knows a phone call to a friend costs 25 cents for the first 3 minutes and 10 cents for each additional minute. The number of minutes you call and the cost of the call are related.

   Identify the variables in this situation: x= __# of min__   y= __cost__

   What is the given information in this problem (find all that apply)?

   y-intercept _____ slope _____ one point (___ , ___) a second point: (___ , ___)

   a. Write an equation for the situation.

   skip

   b. Use your equation to find the cost of a 30-minute call.

More Word Problems in Point-slope form

Sometimes instead of giving a rate, a word problem gives two relationships at different points in time between variables. This kind of problem is giving you two points. You must find the slope and then use one of the points to write an equation.

1. Suppose a 5-minute overseas call costs $5.91 and a 10-minute call costs $10.86. The cost of the call and the length of the call are related.

Identify the variables in this situation: x= # of min   y= cost

What is the given information in this problem (find all that apply)?

y-intercept _____ slope _____ one point (5 , 5.91) a second point: (10 , 10.86)

a. What is the cost y of a call of x minutes in duration? (Assume this is a constant-increase situation)

$$m = \frac{10.86 - 5.91}{10 - 5} = \frac{4.95}{5} = .99$$

$$y - 5.91 = .99(x - 5)$$
$$y - 5.91 = .99x - 4.95$$
$$y = .99x + .96$$

b. How long can you talk on the phone if you have $12 to spend?

$$12 = .99x + .96$$
$$11.04 = .99x \qquad x = 11.15 \text{ min}$$

MORE WORD PROBLEM PRACTICE:

1. Nick is given $50 to spend on a vacation. He decides to spend $5 a day. The amount Nick has left and the number of days are related.

Identify the variables in this situation: x= # of days   y= amt $ left

What is the given information in this problem (find all that apply)?

y-intercept 50 slope -5 one point (/ ,) a second point: (/ ,)

a. Write an equation relating x and y.

$$y = -5x + 50$$

b. Use your equation to find out when Nick will have $15 left.

$$15 = -5x + 50$$
$$-35 = -5x$$
$$x = 7 \text{ days}$$

2. Julio plans a diet to gain 0.2 kg a day. After 14 days he weighs 40 kg. The number days he diets and his weight are related.

Identify the variables in this situation: x= _# of days_ y= _weight_

What is the given information in this problem (find all that apply)?
y-intercept _____ slope _0.2_ one point (_14_ , _40_) a second point: (_____ , _____)

a. Write an equation relating Julio's weight , w, to the number of days, d, on his diet.

$$y-40 = .2(x-14)$$
$$y-40 = .2x - 2.8$$
$$y = .2x + 37.2$$

b. How long will it take Julio to reach his goal weight of 50 kg?

$$50 = .2x + 37.2$$
$$12.8 = .2x$$
$$x = 64 \ kg$$

3. A plane loses altitude at the rate of 5 meters per second. It begins with an altitude of 8500 meters. The plane's altitude is a function of the number of seconds that pass.

Identify the variables in this situation: x= _# of seconds_ y= _altitude_

What is the given information in this problem (find all that apply)?
y-intercept _8500_ slope _-5_ one point (_____) a second point: (_____ , _____)

a. Write an equation modeling this situation.

$$y = -5x + 8500$$

b. Use your equation to find out how much time will pass before the plane will land (hint: what is the altitude when the plane lands?)

$$0 = -5x + 8500$$
$$-8500 = -5x$$
$$x = 1700 \ seconds$$

HMWK Key   Key

Cricket Problem: Based on information in *Deep River Jim's Wilderness Trailbook*, the rate at which crickets chirp is a linear function of temperature. At 59°F they make 76 chirps per minute, and at 65°F they make 100 chirps per minute.

a. Write the particular equation expressing chirping rate (c) in terms of temperature (t).   Chirps depend on temp

(t, c) $(59, 76)$ $(65, 100)$

$$\frac{100-76}{65-59} = \frac{24}{6} \quad 4 \text{ cmp}/°$$

$C - 100 = 4(t - 65)$
$C - 100 = 4t - 260$
$C = 4t - 160$

b. Predict the chirping rate for 90°F.

$C = 4(90) - 160$
$C = 360 - 160$
$C = 200 \text{ c/m}$

c. How warm is it if you count 120 chirps per minute?

$120 = 4t - 160$
$280 = 4t$
$t = 70°$

d. Find the chirping-rate intercept. What does it tell you about the real world?

$(t, c) \quad -160$

At zero° negative chirping "unrealistic"

e. Calculate the temperature-intercept. What does this number represent in the real world?
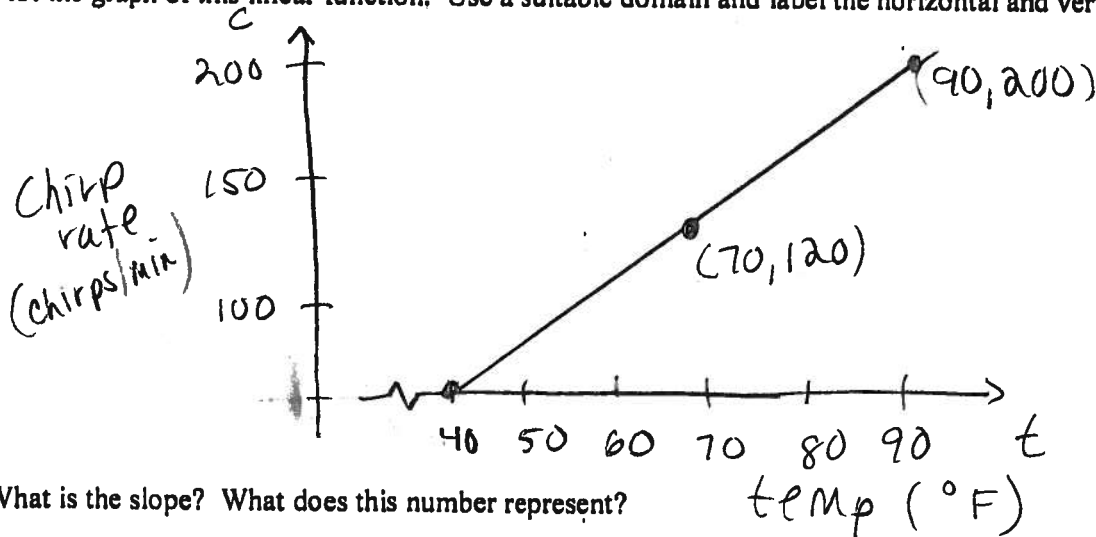
$0 = 4t - 160$
$160 = 4t \quad t = 40$
$(40, 0)$
at 40° crickets don't chirp

f. Plot the graph of this linear function. Use a suitable domain and label the horizontal and vertical axis.

g. What is the slope? What does this number represent?

4 chirp/min/°

# of chirps per minute
4 chirps per 1°F