

tion 9.3: Steps for graphing rational functions

1) Reduce the function, if possible, by factoring both the numerator and the denominator and dividing out any common factors.

- If there is a common factor, it indicates a hole (hiatus point) in the graph.
 - Solve this factor for x . Substitute the x value into the reduced function to find y .
 - This (x,y) point is a hole in the graph.

Use the reduced function for the following steps:

2) Find the vertical asymptote(s).

- Solve the denominator for x .
- * The graph will never cross a vertical asymptote.

3) Find the horizontal asymptote.

- n : degree of numerator
- d : degree of denominator
- $n < d$: horizontal asymptote at $y = 0$
- $n = d$: horizontal asymptote at $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$
- $n > d$: no horizontal asymptote

4) To graph, first plot the hole and the asymptotes. To determine the direction of the curve of the graph in any section, graph the function in your calculator.

x	y
-5	2.5
-4	2.6
-3	error
-2	4
-1	error
0	0

x	y
1	1
2	
3	1.5
4	
5	
6	
7	1.75

$$1) y = \frac{2x^2 + 6x}{x^2 + 4x + 3} = \frac{2x(x+3)}{(x+3)(x+1)}$$

hole @ $(-3, 3)$ VA: $x = -1$

$$y = \frac{2x}{x+1}$$

$$y = \frac{2(-3)}{-3+1} = \frac{-6}{-2} = 3$$

2)

$$y = \frac{2x}{x+1}$$

$x = -1 = \text{vertical asymptote}$

$$3) y = \frac{2x^2 + 6x}{x^2 + 4x + 3}$$

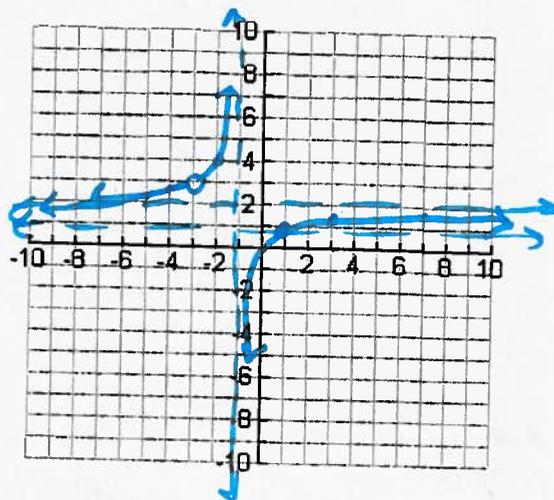
degree of $n = 2$

degree of $d = 2$

$n = d$ horizontal asymptote

$$@ y = \frac{2}{1} = 2$$

$$y = 2$$



Putting it all together:

1) $f(x) = \frac{3}{x-3}$

Hole?

No

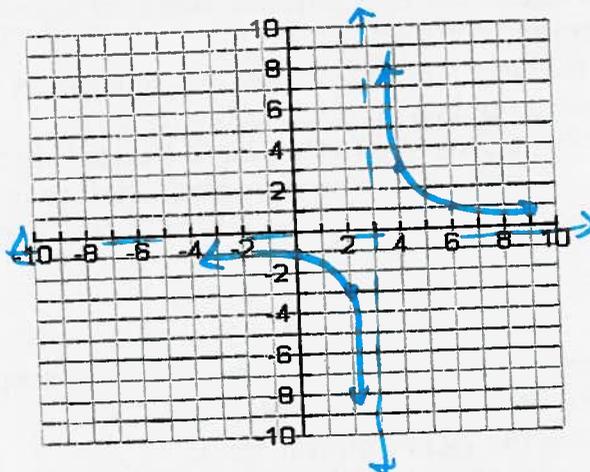
Vertical asymptote

$x=3$

Horizontal asymptote

$n < d \rightarrow y=0$

x	y
0	-1
2	-3
3	error
4	3
6	1



2) $f(x) = \frac{(x-3)(x+2)}{(x-3)(x-1)}$

Hole?

$(3, 2.5)$

$f(x) = \frac{x+2}{x-1}$
 $f(3) = \frac{5}{2}$

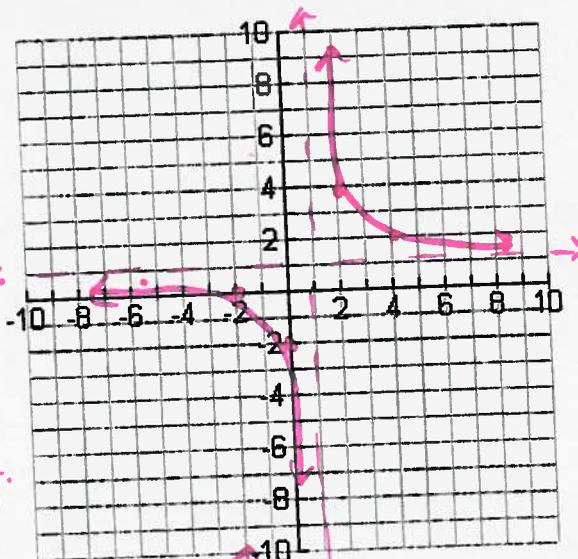
Vertical asymptote

$x=1$

Horizontal asymptote

$n = d \rightarrow y = \frac{1}{1} = 1$

x	y
-5	.5
-2	0
0	2
1	error
2	4
3	undef.
4	2
7	1.5



3) $f(x) = \frac{2x-8}{x+3} = \frac{2(x-4)}{x+3}$

Hole?

No

Vertical asymptote

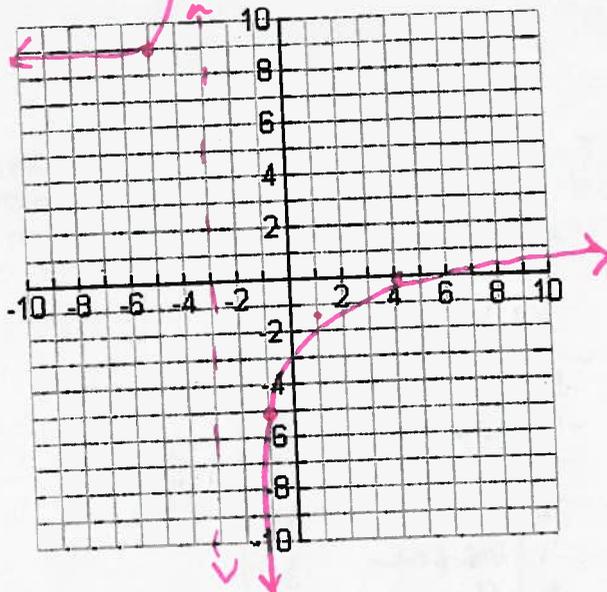
$x=-3$

Horizontal asymptote

$n > d$

No horizontal asymptotes

x	y
-5	9
-4	16
-3	undef
-2	-12
-1	-5
1	-1.5
4	0
11	1



$$4) y = \frac{4x}{x^3 - 4x}$$

$$\frac{4x}{x(x^2-4)} = \frac{4x}{x(x-2)(x+2)}$$

$x=0$
Hole?

~~(0,0)~~ (0,-1)

$$\frac{4}{(x-2)(x+2)}$$

Vertical asymptote

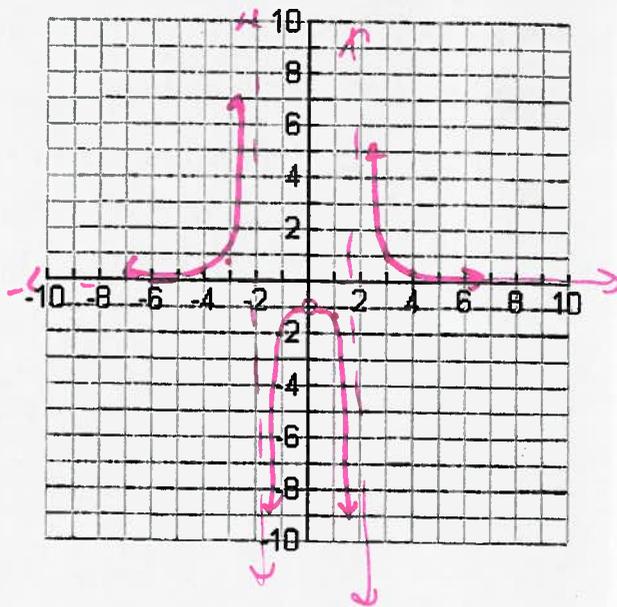
$$x=2$$

$$x=-2$$

Horizontal asymptote

$n < d$ $y=0$

x	y
-4	$\frac{1}{3}$
-3	.8
-2	undef
-1.5	-2.29
-1	$-\frac{1}{3}$
-.5	-1.067
0	undef (hole)
.5	-1.067
1	-1.3
1.5	-2.29
$\frac{3}{4}$.8
4	$\frac{1}{3}$



Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

$$1) f(x) = \frac{1}{3x^2 + 3x - 18}$$

$$2) f(x) = \frac{x-2}{x-4}$$

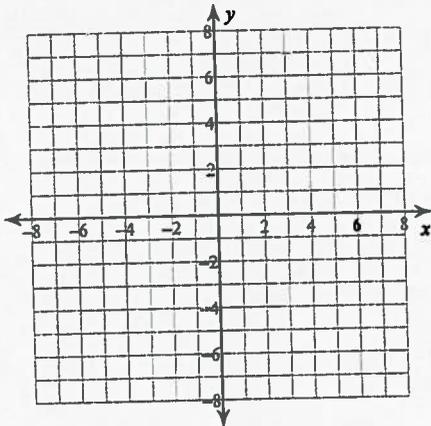
$$3) f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$$

$$4) f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$$

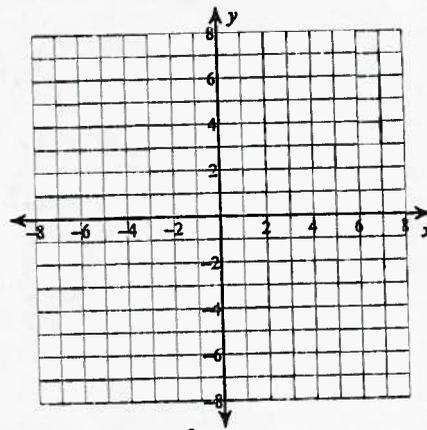
Answers on
next page. ☺

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

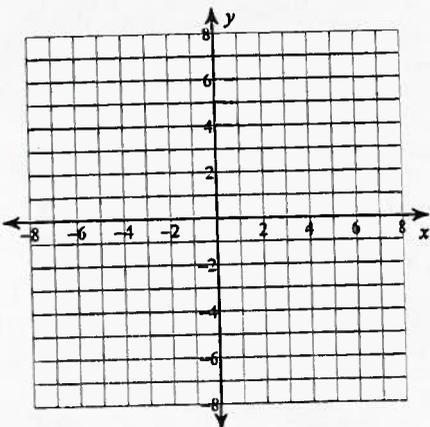
$$5) f(x) = -\frac{4}{x^2 - 3x}$$



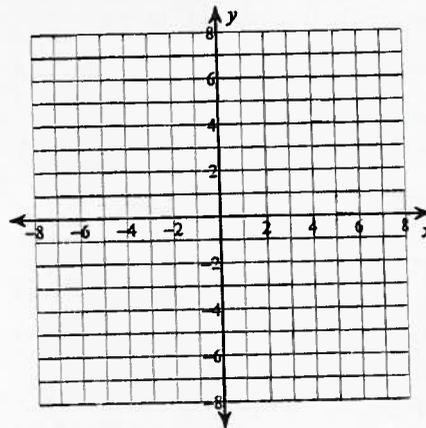
$$6) f(x) = \frac{x-4}{-4x-16}$$



$$7) f(x) = \frac{x+4}{-2x-6}$$



$$8) f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$$



Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$ Discontinuities: -3, 2
 Vertical Asym.: $x = -3, x = 2$
 Holes: None
 Horz. Asym.: $y = 0$
 X-intercepts: None

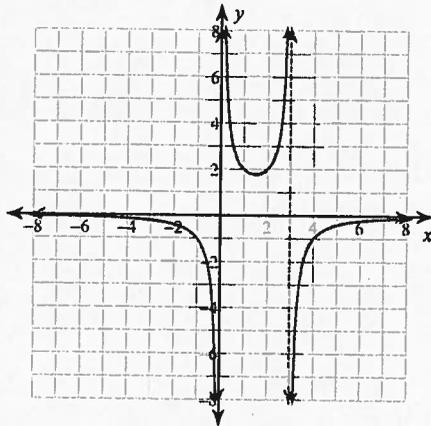
2) $f(x) = \frac{x-2}{x-4}$ Discontinuities: 4
 Vertical Asym.: $x = 4$
 Holes: None
 Horz. Asym.: $y = 1$
 X-intercepts: 2

3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$ Discontinuities: 2, -3
 Vertical Asym.: $x = 2, x = -3$
 Holes: None
 Horz. Asym.: None
 X-intercepts: 0, -2, 3

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$ Discontinuities: -1, -3
 Vertical Asym.: $x = -1$
 Holes: $x = -3$
 Horz. Asym.: $y = -\frac{1}{4}$
 X-intercepts: 2

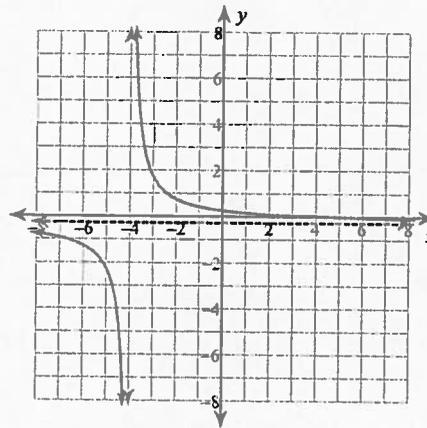
Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

5) $f(x) = -\frac{4}{x^2 - 3x}$



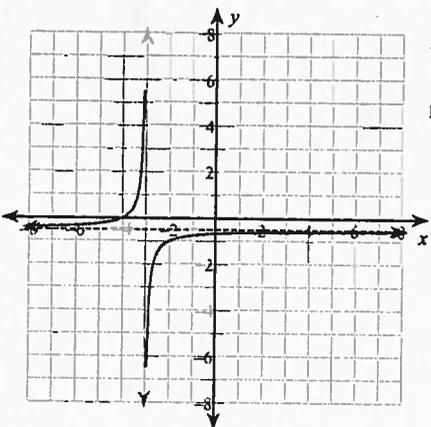
Discontinuities: 0, 3
 Vertical Asym.: $x = 0, x = 3$
 Holes: None
 Horz. Asym.: $y = 0$

6) $f(x) = \frac{x-4}{-4x-16}$



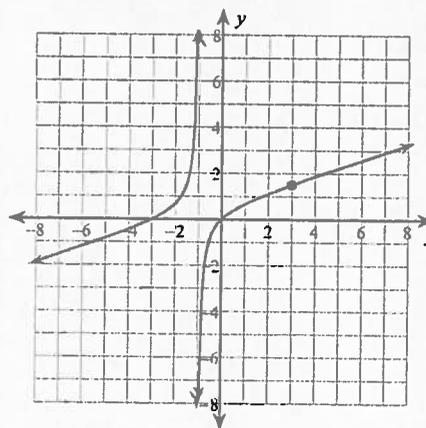
Discontinuities: -4
 Vertical Asym.: $x = -4$
 Holes: None
 Horz. Asym.: $y = -\frac{1}{4}$

7) $f(x) = \frac{x+4}{-2x-6}$



Discontinuities: -3
 Vertical Asym.: $x = -3$
 Holes: None
 Horz. Asym.: $y = -\frac{1}{2}$

8) $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$



Discontinuities: -1, 3
 Vertical Asym.: $x = -1$
 Holes: $x = 3$
 Horz. Asym.: None

