Station 1

1. Divide $(9x^3 - 48x^2 + 13x + 3)$ by $(x - 5)$ using long division.

$$
\begin{array}{r}
9x^2 - 3x - 2 \\
x-5 \enclose{longdiv}{9x^3 - 48x^2 + 13x + 3} \\
-(9x^3 - 45x^2) \\
\hline
-3x^2 + 13x \\
-(-3x^2 + 15x) \\
\hline
-2x + 3 \\
-(-2x + 10) \\
\hline
-7
\end{array}
$$

$\boxed{9x^2 - 3x - 2 \quad R{:}\,-7}$

$\frac{9x^3}{x} = 9x^2$

$9x^2(x-5) = 9x^3 - 45x^2$

$\frac{-3x^2}{x} = -3x$

$-3x(x-5) = -3x^2 + 15x$

$\frac{-2x}{x} = -2$

$-2(x-5) = -2x + 10$

2. Use long division and the given factor $(x - 4)$ to completely factor $(x^3 - 37x + 84)$. State all the zeros of the polynomial function.

$$
\begin{array}{r}
x^2 + 4x - 21 \\
x-4 \enclose{longdiv}{x^3 + 0x^2 - 37x + 84} \\
-(x^3 - 4x^2) \\
\hline
4x^2 - 37x \\
-(4x^2 - 16x) \\
\hline
-21x + 84 \\
-(-21x + 84) \\
\hline
0
\end{array}
$$

$\frac{x^3}{x} = x^2$

$x^2(x-4) = x^3 - 4x^2$

$\frac{4x^2}{x} = 4x$

$4x(x-4) = 4x^2 - 16x$

$\frac{-21x}{x} = -21$

$-21(x-4) = -21x + 84$

$x^2 + 4x - 21$

$(x+7)(x-3)$

Factors of $x^3 - 37x + 84$

$(x+7)(x-3)(x-4)$

zeros: $-7, 3,$ and 4

Station 2

Solve each equation below:

1. $2x^3 + 2 = 0$

Graphing:
zero @ -1

$$-1 \,|\; 2 \quad 0 \quad 0 \quad 2$$
$$\quad -2 \quad 2 \quad -2$$
$$2 \quad -2 \quad 2 \quad \lfloor 0$$

$2x^2 - 2x + 2 = 0$

$x = \dfrac{2 \pm \sqrt{4-16}}{4}$

$= \dfrac{2 \pm \sqrt{-12}}{4} = \dfrac{2 \pm 2i\sqrt{3}}{4}$

$\boxed{x = \dfrac{1 \pm i\sqrt{3}}{2}}$ $\boxed{x = -1}$

Factoring:

$2(x^3+1) = 0$

$2(x+1)(x^2-x+1) = 0$

$x+1=0$ $\quad$ $x^2-x+1=0$

$\boxed{x=-1}$ $\quad$ $x = \dfrac{1 \pm \sqrt{1-4}}{2}$

$x = \dfrac{1 \pm \sqrt{-3}}{2}$

$\boxed{x = \dfrac{1 \pm i\sqrt{3}}{2}}$

2. $x^3 + 125 = 0$

Graphing: zero @ -5

$$-5 \,|\; 1 \quad 0 \quad 0 \quad 125$$
$$\quad -5 \quad 25 \quad -125$$
$$1 \quad -5 \quad 25 \quad \lfloor 0$$

$x^2 - 5x + 25 = 0$

$\boxed{x = \dfrac{5 \pm 5i\sqrt{3}}{2}}$ $\boxed{x=-5}$

Factoring:

$(x+5)(x^2-5x+25) = 0$

$\boxed{x=-5}$ $\quad$ $\boxed{x = \dfrac{5 \pm 5i\sqrt{3}}{2}}$

3. $x^4 - 12x^2 - 64 = 0$

Graphing:
zeros @ -4 and 4

$$-4 \,|\; 1 \quad 0 \quad -12 \quad 0 \quad -64$$
$$\quad -4 \quad 16 \quad 16 \quad 64$$
$$1 \quad -4 \quad 4 \quad 16 \quad \lfloor 0$$

$x^3 - 4x^2 + 4x + 16$

$$4 \,|\; 1 \quad -4 \quad 4 \quad 16$$
$$\quad 4 \quad 0 \quad 16$$
$$1 \quad 0 \quad 4 \quad \lfloor 0$$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\boxed{x = \pm 2i}$

$\boxed{x = \pm 4}$

Factoring:

$(x^2-16)(x^2+4) = 0$

$x^2-16=0$ $\quad$ $x^2+4=0$

$x^2=16$ $\quad$ $x^2=-4$

$\boxed{x=\pm 4}$ $\quad$ $\boxed{x = \pm 2i}$

4. $x^4 - 3x^2 - 28 = 0$

Graphing: only visable zeros are not integers

Factoring:

$(x^2-7)(x^2+4) = 0$

$x^2-7=0$ $\quad$ $x^2+4=0$

$x^2=7$ $\quad$ $x^2=-4$

$\boxed{x=\pm\sqrt{7}}$ $\quad$ $\boxed{x = \pm 2i}$

Station 3

1. The width of a box is 2 m less than the length. The height is 1 m less than the length. The volume of the box is 60m³. Find the length of the box.

x = length

height = $x-1$

width = $x-2$

$V = x(x-1)(x-2)$

$V = x^3 - 3x^2 + 2x$

$V = 60$

$60 = x^3 - 3x^2 + 2x$

$y = x^3 - 3x^2 + 2x - 60$

Graph:

zero @ $x=5$

Therefore length is 5m

2. The product of three consecutive integers, n - 1, n and n + 1, is 210. Write and solve an equation to find each number.

$(n-1)(n)(n+1) = 210$

$n^3 - n = 210$

$y = n^3 - n - 210$

Graph!

zero @ 6

$n = 6$

$n - 1 = 5$

$n + 1 = 7$

integers are 5, 6 and 7

Station 4

1. **Find the zeros and rewrite the polynomial function in factored form:**

$y = x^3 - 2x^2 - 5x + 6$

Graph: zeros at -2, 1 and 3

$y = x^3 - 2x^2 - 5x + 6$

$y = (x+2)(x-1)(x-3)$

2. **Find the roots of each equation:**

a. $x^3 - 2x^2 - 5x + 10 = 0$

Zero at $x=2$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$x^2 - 5 = 0$

$x^2 = 5$

$\boxed{x = \pm\sqrt{5}}$ $\boxed{x = 2}$

b. $45x^3 + 93x^2 - 12 = 0$

Zero @ $x = -2$

$$\begin{array}{r|rrrr} -2 & 45 & 93 & 0 & -12 \\ & & -90 & -6 & 12 \\ \hline & 45 & 3 & -6 & 0 \end{array}$$

$45x^2 + 3x - 6 = 0$

$x = \frac{-3 \pm \sqrt{1089}}{90}$

$x = \frac{-3 \pm 33}{90}$, $\frac{30}{90} = \frac{1}{3}$, $\frac{-36}{90} = \frac{-2}{5}$

$3(15x^2 + x - 2) = 0$

$3(5x+2)(3x-1) = 0$

$\boxed{x = -\frac{2}{5} \quad x = \frac{1}{3} \quad x = -2}$

c. $9x^4 + 3x^3 - 30x^2 + 6x + 12 = 0$

Zeros @ -2 and 1

$$\begin{array}{r|rrrrr} -2 & 9 & 3 & -30 & 6 & 12 \\ & & -18 & 30 & 0 & -12 \\ \hline & 9 & -15 & 0 & 6 & 0 \end{array}$$

$9x^3 - 15x^2 + 6$

$$\begin{array}{r|rrrr} 1 & 9 & -15 & 0 & 6 \\ & & 9 & -6 & -6 \\ \hline & 9 & -6 & -6 & 0 \end{array}$$

$9x^2 - 6x - 6 = 0$

$x = \frac{6 \pm \sqrt{252}}{18} = \frac{6 \pm 6\sqrt{7}}{18} = \frac{1 \pm \sqrt{7}}{3}$

$\boxed{x = -2 \quad x = 1 \quad x = \frac{1 \pm \sqrt{7}}{3}}$

Station 5

1. Find a third-degree polynomial equation with integer coefficients that has roots 8 and 3i.

$8, 3i$ and $-3i$

$(x-8)(x-3i)(x+3i)$

$(x-8)(x^2+3ix-3ix-9i^2)$

$(x-8)(x^2+9)$

$x^3+9x-8x^2-72$

$\underline{x^3-8x^2+9x-72=0}$

2. Find a fourth-degree polynomial equation with integer coefficients that has roots, 3 + i and -2i.

$3+i, 3-i, -2i, 2i$

$(x-(3+i))(x-(3-i))(x+2i)(x-2i)$

$(x-3-i)(x-3+i)$

$x^2-3x+\cancel{ix}$

$-3x+9-\cancel{3i}$

$-\cancel{ix}+\cancel{3i}-i^2$

$x^2-6x+9+1$

$x^2-6x+10$

$(x^2-2ix+2ix-4i^2)$

(x^2+4)

$(x^2-6x+10)(x^2+4)$

$x^4+4x^2-6x^3-24x$

$+10x^2+40$

$\underline{x^4-6x^3+14x^2-24x+40=0}$

Extra

1. **What is the largest number of real roots that a 7^{th} degree polynomial could have? What is the smallest number?**

a seventh degree polynomial has at least 1 real root and at most 7 real roots

2. **Write an expression that represents the width of a rectangle with length x + 5 and an area of $x^3 + 12x^2 + 47x + 60$.**

$A = l \cdot w$

$w = \frac{A}{l}$

$$\begin{array}{r|rrrr} -5 & 1 & 12 & 47 & 60 \\ & & -5 & -35 & -60 \\ \hline & 1 & 7 & 12 & \boxed{0} \end{array}$$

$w = x^2 + 7x + 12$

3. **One root of the equation $x^3 + x^2 - 2 = 0$ is $x = 1$. How many roots are there? What are all the roots for this polynomial equation?**

degree 3 = 3 roots

$$\begin{array}{r|rrrr} 1 & 1 & 1 & 0 & -2 \\ & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & \boxed{0} \end{array}$$

$x^2 + 2x + 2 = 0$

$x = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$

$\boxed{x = -1 \pm i}$

4. **Determine whether the binomial (x - 4) is a factor of the polynomial $P(x) = 5x^3 - 20x^2 - 5x + 20$.**

$$\begin{array}{r|rrrr} 4 & 5 & -20 & -5 & 20 \\ & & 20 & 0 & -20 \\ \hline & 5 & 0 & -5 & \boxed{0} \end{array}$$

Since the remainder is zero x-4 is a factor of $5x^3 - 20x^2 - 5x + 20$